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ABSTRACT

This paper aims at determining pairs of rectangles such that, in each pair, the sum of their areas is represented by a Gopa - Vidh number. Also, the number of primitive and non-primitive rectangles for each Gopa - Vidh number is given.

KEYWORDS: Pairs of rectangles, Area, Gopa - Vidh number.

1. INTRODUCTION

Any sequence of numbers represented by a mathematical function may be considered as pattern. In fact, mathematics can be considered as a characterization of patterns. For clear understanding, any regularity that can be illustrated by a scientific theory is a pattern. In other words, a pattern is a group of numbers, shapes or objects that follow a rule. A careful observer of patterns may note that there is a one to one correspondence between the polygonal numbers and the number of sides of the polygon. Apart from the above patterns we have some more fascinating patterns of numbers namely Nasty number, Dhuruva numbers and Jarasandha numbers. For illustrations, one may refer [1- 13].

2. DEFINITION

Gopa - Vidh Number: Let N be a non-zero positive integer. Let a represent the sum of the digits in N^2 . If N^2 is a square multiple of a , then, the integer N is referred as Gopa – Vidh number.

3. METHOD OF ANALYSIS

Let $R_1(x, y)$ and $R_2(z, w)$ be two distinct rectangles whose corresponding areas are A_1, A_2 .

Consider

$$A_1 + A_2 = 20, \text{ a Gopa - Vidh number}$$

That is,

$$xy + zw = 20 \tag{1}$$

Let q, r, s be three non-zero distinct positive integers and $r > s$.

Introduction of the linear transformations

$$x = s, y = 2q + s, z = r - s, w = r + s \tag{2}$$

in (1) leads to

$$r^2 = 20 - 2qs \tag{3}$$

Solving (3) for q, r, s and using (2), the corresponding values of rectangles R_1 and R_2 are obtained and presented in Table: 1 below:

Table: 1 Rectangles

Gopa-Vidh Number	R_1	R_2	Observations	
			Primitive	Non-Primitive
20	(1, 5)	(3, 5)	R_1, R_2	
	(2, 4)	(2, 6)		R_1, R_2

Some other numerical examples of Gopa - Vidh numbers are presented in Table: 2 below:

Table: 2 Rectangles

Gopa-Vidh number	R_1	R_2	Observations		Remarks
			Primitive	Non-Primitive	
45	(1, 21)	(4, 6)	R_1	R_2	Total number of Primitive rectangles =2 Total number of non-Primitive rectangles =2
	(2, 12)	(3, 7)	R_2	R_1	
	(1, 13)	(5, 7)	R_1, R_2		
51	(1, 3)	(6, 8)	R_1	R_2	Total number of Primitive rectangles =3 Total number of non-Primitive rectangles =3
	(1, 27)	(4, 6)	R_1	R_2	
	(1, 43)	(2, 4)	R_1	R_2	
60	(1, 25)	(5, 7)	R_1, R_2		Total number of Primitive rectangles =7 Total number of non-Primitive rectangles =7
	(1, 45)	(3, 5)	R_1, R_2		
	(1, 57)	(1, 3)	R_1, R_2		
	(4, 10)	(2, 10)		R_1, R_2	
	(3, 11)	(3, 9)	R_1	R_2	
	(2, 14)	(4, 8)		R_1, R_2	
	(2, 24)	(2, 6)		R_1, R_2	
100	(6, 12)	(2, 14)		R_1, R_2	Total number of Primitive rectangles =11 Total number of non-Primitive rectangles =11
	(3, 15)	(5, 11)	R_2	R_1	
	(4, 20)	(2, 10)		R_1, R_2	
	(2, 20)	(6, 10)		R_1, R_2	
	(3, 31)	(1, 7)	R_1, R_2		

	(2, 34)	(4, 8)		R_1, R_2	
	(1, 37)	(7, 9)	R_1, R_2		
	(2, 4)	(2, 6)		R_1, R_2	
	(1, 65)	(5, 7)	R_1, R_2		
	(1, 85)	(3, 5)	R_1, R_2		
	(1, 97)	(1, 3)	R_1, R_2		
105	(6, 10)	(3, 15)		R_1, R_2	Total number of Primitive rectangles =16 Total number of non-Primitive rectangles =18
	(2, 14)	(7, 11)	R_2	R_1	
	(2, 30)	(5, 9)	R_2	R_1	
	(2, 42)	(3, 7)	R_2	R_1	
	(2, 50)	(1, 5)	R_2	R_1	
	(4, 10)	(5, 13)	R_2	R_1	
	(3, 11)	(6, 12)	R_1	R_2	
	(4, 18)	(3, 11)	R_2	R_1	
	(4, 24)	(1, 9)	R_2	R_1	
	(4, 18)	(3, 11)	R_2	R_1	
	(4, 24)	(1, 9)	R_2	R_1	
	(1, 25)	(8, 10)	R_1	R_2	
	(2, 30)	(5, 9)	R_2	R_1	
	(2, 50)	(1, 5)	R_2	R_1	
	(1, 57)	(6, 8)	R_1	R_2	
	(1, 81)	(4, 6)	R_1	R_2	
	(1, 97)	(2, 4)	R_1	R_2	
200	(1, 5)	(13, 15)	R_1, R_2		Total number of Primitive rectangles =16 Total number of non-Primitive rectangles =18
	(2, 4)	(12, 16)		R_1, R_2	

	(2 , 30)	(10 , 14)		R_1, R_2	
	(2 , 52)	(8 , 12)		R_1, R_2	
	(2 , 70)	(6 , 10)		R_1, R_2	
	(2 , 84)	(4 , 8)		R_1, R_2	
	(2 , 94)	(2 , 6)		R_1, R_2	
	(7 , 15)	(5 , 19)	R_1, R_2		
	(4 , 18)	(8 , 16)		R_1, R_2	
	(4 , 38)	(4 , 12)		R_1, R_2	
	(5 , 25)	(5 , 15)		R_1, R_2	
	(1 , 57)	(11 , 13)	R_1, R_2		
	(1 , 101)	(9 , 11)	R_1, R_2		
	(1 , 137)	(7 , 9)	R_1, R_2		
	(1 , 165)	(5 , 7)	R_1, R_2		
	(1 , 185)	(3 , 5)	R_1, R_2		
	(1 , 197)	(1 , 3)	R_1, R_2		
625	(1 , 97)	(22 , 24)	R_1	R_2	Total number of Primitive rectangles =58 Total number of non-Primitive rectangles =66
	(1 , 185)	(20 , 22)	R_1	R_2	
	(1 , 265)	(18 , 20)	R_1	R_2	
	(1 , 337)	(16 , 18)	R_1	R_2	
	(1 , 401)	(14 , 16)	R_1	R_2	
	(1 , 457)	(12 , 14)	R_1	R_2	
	(1 , 505)	(10 , 12)	R_1	R_2	
	(1 , 545)	(8 , 10)	R_1	R_2	
	(1 , 577)	(6 , 8)	R_1	R_2	
	(1 , 601)	(4 , 6)	R_1	R_2	



(1 , 617)	(2 , 4)	R_1	R_2
(2 , 50)	(21 , 25)	R_2	R_1
(2 , 94)	(19 , 23)	R_2	R_1
(2 , 134)	(17 , 21)	R_2	R_1
(2 , 170)	(15 , 19)	R_2	R_1
(2 , 402)	(13 , 17)	R_2	R_1
(2 , 230)	(11 , 15)	R_2	R_1
(2 , 254)	(9 , 13)	R_2	R_1
(2 , 274)	(7 , 11)	R_2	R_1
(2 , 290)	(5 , 9)	R_2	R_1
(2 , 302)	(3 , 7)	R_2	R_1
(2 , 310)	(1 , 5)	R_2	R_1
(16 , 22)	(7 , 39)	R_2	R_1
(3 , 35)	(20 , 26)	R_1	R_2
(3 , 91)	(16 , 22)	R_1	R_2
(3 , 115)	(14 , 20)	R_1	R_2
(3 , 155)	(10 , 16)	R_1	R_2
(3 , 171)	(8 , 14)		R_1, R_2
(3 , 195)	(4 , 10)		R_1, R_2
(3 , 203)	(2 , 8)	R_1	R_2
(4 , 28)	(19 , 27)	R_2	R_1
(12 , 20)	(11 , 35)	R_2	R_1
(4 , 50)	(17 , 25)	R_2	R_1
(4 , 70)	(15 , 23)	R_2	R_1
(4 , 88)	(13 , 21)	R_2	R_1



	(4 , 104)	(11 , 19)	R_2	R_1
	(4 , 118)	(9 , 17)	R_2	R_1
	(4 , 130)	(7 , 15)	R_2	R_1
	(4 , 140)	(5 , 13)	R_2	R_1
	(4 , 148)	(3 , 11)	R_2	R_1
	(4 , 154)	(1 , 9)	R_2	R_1
	(5 , 85)	(10 , 20)		R_1, R_2
	(6 , 24)	(17 , 29)	R_2	R_1
	(6 , 50)	(13 , 25)	R_2	R_1
	(6 , 62)	(11 , 23)	R_2	R_1
	(6 , 82)	(7 , 19)	R_2	R_1
	(6 , 90)	(5 , 17)	R_2	R_1
	(6 , 102)	(1 , 13)	R_2	R_1
	(7 , 55)	(10 , 24)	R_1	R_2
	(7 , 79)	(4 , 18)	R_1	R_2
	(8 , 20)	(15 , 31)	R_2	R_1
	(6 , 22)	(17 , 29)	R_2	R_1
	(8 , 50)	(9 , 25)	R_2	R_1
	(8 , 58)	(7 , 23)	R_2	R_1
	(8 , 76)	(1 , 17)	R_2	R_1
	(9 , 65)	(2 , 20)	R_1	R_2
	(10 , 50)	(5 , 25)		R_1, R_2
	(11 , 35)	(8 , 30)	R_1	R_2
	(12 , 34)	(7 , 31)	R_2	R_1
	(12 , 50)	(1 , 25)	R_2	R_1

	(12 , 40)	(5 , 29)	R_2	R_1	
	(14 , 38)	(3 , 31)	R_2	R_1	
1010	(1 , 111)	(29 , 31)	R_1, R_2		Total number of Primitive rectangles =42 Total number of non-Primitive rectangles =4
	(1 , 227)	(27 , 29)	R_1, R_2		
	(1 , 335)	(25 , 27)	R_1, R_2		
	(1 , 435)	(23 , 25)	R_1, R_2		
	(1 , 527)	(21 , 23)	R_1, R_2		
	(1 , 611)	(19 , 21)	R_1, R_2		
	(1 , 687)	(17 , 19)	R_1, R_2		
	(1 , 755)	(15 , 17)	R_1, R_2		
	(1 , 815)	(13 , 15)	R_1, R_2		
	(1 , 867)	(11 , 13)	R_1, R_2		
	(1 , 911)	(9 , 11)	R_1, R_2		
	(1 , 947)	(7 , 9)	R_1, R_2		
	(1 , 975)	(5 , 7)	R_1, R_2		
	(1 , 995)	(3 , 5)	R_1, R_2		
	(1 , 1007)	(1 , 3)	R_1, R_2		
	(5 , 27)	(25 , 35)	R_1	R_2	
	(11 , 21)	(19 , 41)	R_1, R_2		
	(5 , 127)	(15 , 25)	R_1	R_2	
	(5 , 187)	(5 , 15)	R_1	R_2	
	(7 , 69)	(17 , 31)	R_1, R_2		
(7 , 105)	(11 , 25)	R_2	R_1		
(7 , 137)	(3 , 17)	R_1, R_2			
(13 , 71)	(3 , 29)	R_1, R_2			

1100	(1, 77)	(31, 33)	R_1, R_2		Total number of Primitive rectangles =39 Total number of non-Primitive rectangles =53
	(1, 201)	(29, 31)	R_1, R_2		
	(1, 317)	(27, 29)	R_1, R_2		
	(1, 425)	(25, 27)	R_1, R_2		
	(1, 525)	(23, 25)	R_1, R_2		
	(1, 617)	(21, 23)	R_1, R_2		
	(1, 701)	(19, 21)	R_1, R_2		
	(1, 777)	(17, 19)	R_1, R_2		
	(1, 845)	(15, 17)	R_1, R_2		
	(1, 905)	(13, 15)	R_1, R_2		
	(1, 957)	(11, 13)	R_1, R_2		
	(1, 1001)	(9, 11)	R_1, R_2		
	(1, 1037)	(7, 9)	R_1, R_2		
	(1, 1065)	(5, 7)	R_1, R_2		
	(1, 1085)	(3, 5)	R_1, R_2		
	(1, 1097)	(1, 3)	R_1, R_2		
	(2, 40)	(30, 34)		R_1, R_2	
	(2, 102)	(28, 32)		R_1, R_2	
	(19, 23)	(13, 51)	R_1, R_2		
	(2, 160)	(26, 30)		R_1, R_2	
	(2, 214)	(24, 28)		R_1, R_2	
	(2, 264)	(22, 26)		R_1, R_2	
	(2, 310)	(20, 24)		R_1, R_2	
	(2, 352)	(18, 22)		R_1, R_2	
	(2, 390)	(16, 20)		R_1, R_2	



	(2 , 424)	(14 , 18)		R_1, R_2
	(2 , 454)	(12 , 16)		R_1, R_2
	(2 , 480)	(10 , 14)		R_1, R_2
	(2 , 502)	(8 , 12)		R_1, R_2
	(2 , 520)	(6 , 10)		R_1, R_2
	(2 , 534)	(4 , 8)		R_1, R_2
	(2 , 544)	(2 , 6)		R_1, R_2
	(4 , 54)	(26 , 34)		R_1, R_2
	(25 , 33)	(5 , 55)	R_1	R_2
	(4 , 110)	(22 , 30)		R_1, R_2
	(4 , 158)	(18 , 26)		R_1, R_2
	(4 , 198)	(14 , 22)		R_1, R_2
	(4 , 230)	(10 , 18)		R_1, R_2
	(4 , 254)	(6 , 14)		R_1, R_2
	(4 , 270)	(2 , 10)		R_1, R_2
	(5 , 45)	(25 , 35)		R_1, R_2
	(20 , 30)	(10 , 40)		R_1, R_2
	(5 , 145)	(15 , 25)		R_1, R_2
	(5 , 205)	(5 , 15)		R_1, R_2
	(7 , 107)	(13 , 27)	R_1, R_2	
	(7 , 155)	(1 , 15)	R_1, R_2	

4. CONCLUSION

In this paper, an attempt has been made to obtain pairs of rectangles such that, in each pair, the sum of their areas is represented by a Gopa - Vidh number. The readers of this paper may search for pairs of rectangles for other choices of Gopa - Vidh numbers.





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